APPLICATION FOR UNITED STATES PATENT

FOR

WIRELESS SIGNAL PROCESSING METHODS AND APPARATUSES INCLUDING DIRECTIONS OF ARRIVAL ESTIMATION

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Technical Field

The present invention is related to the field of communication in general, and wireless communication, in particular.

BACKGROUND

Advances in integrated circuit, microprocessor, networking, telecommunication and other related technologies have led to wide spread adoption of wireless communication, e.g. mobile wireless "cell" phones. In the case of wireless communication, such as mobile wireless "cell" phones, typically, a mobile wireless "cell" phone (also referred to as a mobile handset), communicates with a "nearby" service station (also referred to as a base station), which relays the communication signals for the mobile handset. The service/base station provides the relay service for all mobile handset in its coverage area (its "cell"). Thus, a service/base station typically receives and processes communication signals from a number of mobile handsets in its "cell" concurrently.

Use of multiple antennas at the service/base station for receiving the communication signals from the mobile handsets have become popular, as it has several advantages in terms of enhancing the capacity and throughput of the wireless communication system. Various signal processing techniques are employed to process the received communication signals, including but not limited to "space-time" processing techniques.

Among the space-time processing techniques, beamforming is one of the promising areas of interest for enhancing the strength of signals received from a desired direction. One known technique is the employment of a known training sequence to estimate the optimum weights (e.g. using least mean square (LMS)), for beamforming to a desirable direction. Other known techniques for

estimating the directions of arrival (DOAs) include employment of the Bartlett processor or the MUSIC (*Mu*ltiple *Si*gnal Classification) technique.

Training has the disadvantage of incurring overhead in the throughput of the system, and convergence may take longer time than the time available to make the determination. The latter techniques require a large number of snapshots of the received signals to provide a good estimate of a correlation matrix reflective of the correlation (or the lack thereof) of the received signals from the independent signal sources.

Additionally, while each communication signal typically has a number of multipaths, due to environmental factors, such as reflection off structures and so forth, these techniques typically estimate the DOA based only on the most dominant multipath of a signal.

BRIEF DESCRIPTION OF THE DRAWINGS

Embodiments of the present invention will be described by way of the accompanying drawings in which like references denote similar elements, and in which:

Figure 1 illustrates a communication environment suitable for practicing the present invention, in accordance with one embodiment;

Figure 2 illustrates a portion of the operational flow for determining the directions of arrival of *J* signals wirelessly transmitted from *J* signal sources, in accordance with one embodiment;

Figure 3 illustrates a portion of the operational flow for determining the directions of arrival of *L* multipaths of *J* signals wirelessly transmitted from *J* signal sources, in accordance with one embodiment;

Figure 4 illustrates a computer system suitable for use to practice one or more aspects of one or more of the signal processing methods of **Fig. 2-3**, in accordance with one embodiment.

DETAILED DESCRIPTION OF EMBODIMENTS OF THE INVENTION

Embodiments of the present invention include but are not limited to methods and apparatuses for determining the direction of arrivals of a number of signals wirelessly transmitted. Embodiments of the present invention also include methods and apparatuses for determining the direction of arrivals of multipaths of a number of signals wirelessly transmitted.

In the following description, various aspects of embodiments of the present invention will be described. However, it will be apparent to those skilled in the art that other embodiments may be practiced with only some or all of the described aspects. For purposes of explanation, specific numbers, materials and configurations are set forth in order to provide a thorough understanding of the embodiments. However, it will be apparent to one skilled in the art that other embodiments may be practiced without the specific details. In other instances, well-known features are omitted or simplified in order not to obscure the description.

Various operations will be described as multiple discrete operations in turn, in a manner that is most helpful in understanding the embodiments, however, the order of description should not be construed as to imply that these operations are necessarily order dependent. In particular, these operations need not be performed in the order of presentation.

The phrase “in one embodiment” is used repeatedly. The phrase generally does not refer to the same embodiment, however, it may. The terms “comprising”, “having” and “including” are synonymous, unless the context dictates otherwise.

Referring now to **Fig. 1** wherein an overview of a communication environment suitable for practicing the present invention, in accordance with one embodiment, is shown. As illustrated, for the embodiment, communication environment **100** includes *J* mobile handsets **102a-102j** and base station **106**, communicatively coupled to each other. Base station **106** relays communication signals for mobile handsets **102a-102j** including receiving signals wirelessly transmitted by the handsets. The received signals, among other things, may be processed and forwarded to another signal processing node (not shown) of a wireless communication service network, to which base station **106** is coupled, and/or to a signal processing node (not shown) of a PSTN (Public Switched Telephone Network).

For the embodiment, base station **106** includes N antennas **108a-108n**, RF unit **110**, and signal processing unit **110**, coupled to each other as shown (RF = Radio Frequency). Antennas **108a-108n** are employed to transmit to, and receive signals from mobile handsets **102a-102j**. Additionally, antennas **108a-108n** may be employed for other purposes. Antennas **108a-108n** may also be referred to as sensors. For the purpose of this application, the two terms are synonymous.

RF unit **110** is employed to down convert RF signals received by antennas **108a-108n** into baseband signals, or up convert baseband signals into RF signals for transmission by antennas **108a-108n**.

Signal processing unit **110** is employed to process the down converted baseband signals, and process outbound signals for up conversion. For the embodiment, signal processing unit **110**, includes in particular DOA Estimation

unit **112** and Beamforming unit **114**, coupled to each other and to RF unit **110** as shown.

In various embodiments, DOA Estimation unit **112** estimates the DOAs of the *J* signals (*J* being an integer), to be described more fully below referencing **Fig. 2**. For these embodiments, Beamforming unit **114** forms the corresponding weighted output signals based at least in part on the DOAs of the *J* signals estimated by DOA Estimation unit **112**.

In other embodiments, DOA Estimation unit **112** estimates the DOAs of the *L* multipaths (*L* being an integer) of the *J* signals, to be described more fully referencing **Fig. 3**. Some embodiments support one of the two methods to be described referencing **Fig. 2** and **3**. Other embodiments support both. For those embodiments supporting both methods illustrated by **Fig. 2-3**, the embodiments may also support configuration of one of the two methods as the current operating method. The configuration may be supported in a static or dynamic approach.

For the latter embodiments, Beamforming unit **114** forms the corresponding output signals based at least in part on the DOAs of the *L* multipaths of the *J* signals estimated by DOA Estimation unit **112**. More specifically, Beamforming unit **114** determines a number of corresponding weights and forms the corresponding output signals based at least in part on weighted contributions of the *L* multipaths of the *J* signals estimated by DOA Estimation unit **112**.

Except for the advantageous manners base station **106** acquires signals **104a-104j**, mobile handsets **102a-102j** and RF units **110** represent a broad range of these elements. A computer system suitable for hosting a software implementation of DOA Estimation unit **112** and Beamforming unit **114** will be

further described below referencing **Fig. 4**. The software implementation may be developed employing any one or more of a number of programming languages. However, the present invention anticipates all or portions of DOA Estimation unit **112** and Beamforming unit **114** may be implemented in hardware, using e.g. one or more Application Specific Integrated Circuit (ASIC) or Field Programmable Logic Devices. One of ordinary skill in the art would be able to do so, based on the description provided herein.

For the remaining descriptions to follow, and for the claims, the following conventions are employed:

- boldface-capital letters, such as **A**, **M**, represent matrices or subspaces,
- boldface-small letters, such as **v**, represent vectors, and
- non-boldface letters, such as "s", represent scalars.

Referring now to **Fig. 2**, a flow diagram illustrating the operation flow of DOA Estimation unit **112** for determining the DOAs of the *J* signals in accordance with one embodiment, is shown. The embodiment assumes the followings:

The *J* independent signals impinge on the N antenna array in J distinct directions $\theta_1,...\theta_J$, where the angles θ_j are measured with respect to the endfire direction.

The output signal vector for a single snapshot of the received signal is given by

(1) $$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)$$

where $\mathbf{x}(t)$ is the $N \times 1$ column vector written as,

(2) $$\mathbf{x}(t) = [x_1(t),...,x_L(t)]^T$$

(3) $\mathbf{A} = [\mathbf{a}(\theta_1),...\mathbf{a}(\theta_J)]$, where $\mathbf{A}$ is the $N \times J$ directional response matrix

(4) $\mathbf{a}(\theta_j) = [1 \quad e^{-ikd\cos\theta_j} \quad . \quad . \quad e^{-i(L-1)kd\cos\theta_j}]^T, \; j = 1,...,J$

are the signal directional vectors. Each $\mathbf{a}(\theta_j)$ vector has the dimension $N \times 1$.

(5) $\mathbf{s}(t) = [s_1(t),...,s_J(t)]^T$, $\mathbf{s}(t)$ is the $J \times 1$ vector that contains the J signals transmitted by J independent sources.

$k = 2\pi f_0 / c$, f_0 being the center frequency, and c being speed of electromagnetic wave, k being the wavelength, and

d is the spacing between antenna elements and t corresponds to the time index..

Thus, $\mathbf{x}(t)$ is the linear combination of *J*-linearly independent Vandermonde vectors, each corresponding to a source direction, and $\mathbf{x}(t)$ may be considered as

(6) $\mathbf{x}(t) = \mathbf{a}(\theta_1)s_1(t) + ... + \mathbf{a}(\theta_J)s_J(t)$

In Equation (3), matrix $\mathbf{A}$ represents a basis that spans a *J*-dimensional subspace, to which $\mathbf{x}(t)$ belongs. Accordingly, $\mathbf{x}(t)$ in its decomposition has one of the directional vectors as its element.

Hence, there exists a correlation such that the intersection of the one-dimensional subspace spanned by $\mathbf{x}(t)$ (output signal vector for a single snapshot) and any of it's *J*-signal directional vectors is non-zero.

Therefore, if θ_i does not correspond to one of true source directions i.e. $\theta_i \notin (\theta_1,...,\theta_J)$, the intersection between $\mathbf{x}(t)$ and $\mathbf{a}(\theta_i)$ will be zero.

Accordingly, the computational problem is formulated as

(7) $\mathbf{x}(t) \cap \mathbf{a}(\theta_i) = 0$, for $\theta_i \notin (\theta_1,...,\theta_J)$

(8) $\mathbf{x}(t) \cap \mathbf{a}(\theta_i) \neq 0$ for $\theta_i \in (\theta_1,...,\theta_J)$

The Nx2-matrix formulated as $\mathbf{D}(t)=[\mathbf{x}(t),\mathbf{a}(\theta_i)]$ will have rank deficiency or close to rank deficiency if $\theta_i \in (\theta_1,...,\theta_J)$.

Hence, in determining the DOA of the *J* signals $\theta_I,...\theta_J$, the process of **Fig. 2** first selects a trial direction for a signal source *j*, $\sim\theta_i$, block **202**.

On selecting a trial direction, the process proceeds to compute

a first coefficient (9a) $r_{11}=\|\mathbf{x}(t)\|$, and

a first orthonormal vector (9b) $\mathbf{q}_1=\dfrac{\mathbf{x}(t)}{r_{11}}$, block **204**.

On computing the first coefficient and the first orthonormal vector, the process proceeds to compute

a second coefficient (10a) $r_{12}=\mathbf{q}_1^H\mathbf{a}(\sim\theta_i)$,

a second orthonormal vector (10b) $\mathbf{q}_2=\mathbf{a}(\sim\theta_i)-r_{12}\mathbf{q}_1$, and

a third coefficient (10c) $r_{22}=\|\mathbf{q}_2^H\mathbf{a}(\sim\theta_i)\|$, block **206**.

Then, the process proceeds to evaluating a function

(11) $B(\theta)=\dfrac{1}{r_{22}}$ for $\sim\theta_i$, and

determines whether the evaluation yields a peak value for the function. If so, the trial direction is considered to be one of the DOAs of the *J* signals.

At block **210**, the process determines whether additional directions are to be determined. If so, the process returns to block **202**, and continues from there as earlier described. If not, the process terminates.

Referring now to **Fig. 3**, a flow diagram illustrating the operation flow of DOA Estimation unit **112** for determining the DOA of *L* coherent multipaths of *J* signals in accordance with one embodiment, is shown. The embodiment includes an initial determination of the DOAs of the *J* signals, and the determined

DOA of the J signals are then employed, along with other information, to determine the DOAs of the L multipaths of the J signals.

The signal corresponding to jth source and L coherent multipaths at the base station is given by

$$(12) \qquad x_j(t)=\sum_{l=1}^{L} R_{jl} e^{i2\pi(f_d \cos\theta_{jl}-f\tau_{jl})} s(t-\tau_{jl})$$

Extending the above equation for N-element antenna array, the following relationship is obtained.

$$(13) \quad \mathbf{x}_j(t)=\sum_{l=1}^{L} \mathbf{v}(\theta_{jl}) R_{jl} e^{i2\pi(f_d cos\theta_{jl}-f\tau_{jl})} s(t-\tau_{jl}),$$

where $\mathbf{v}(\theta_{jl})=[1 \quad e^{\frac{-i2\pi Cos\theta_{jl}}{\lambda}} \quad e^{\frac{i4\pi Cos\theta_{jl}}{\lambda}} \quad \ldots\ldots. \quad e^{\frac{i2\pi(N-1)Cos\theta_{jl}}{\lambda}} \;]^T$ is the array response vector for jl-th multipath component. The notations in the above have the following meanings.

R_{jl} in the signal strength or amplitude of the l'th multipath.

$f_d Cos\theta_{jl}$ is the Doppler shift of the jl'th multipath ,

$\tau_{jl}=r_{jl}/c$ is the time delay of the of the jl-th multipath,

r_{jl} is the range of the jl-th multipath, and

c is the speed of electromagnetic wave,

θ_{jl} is DOA corresponding to jl-th multipath.

L is the number of dominant multipaths for j-th source.

Under the assumption that multipath delay spread

$$T = max(\tau_{jl}) - min(\tau_{jl}) << 1/B,$$

where B is the bandwidth of the signal,

$s_j(t)$, the base band signal of the jth source can be modeled as a narrowband signal, that is, $s(t-\tau_{jl}) \approx s(t-\tau_0)$, where $\tau_0 \in [min(\tau_i), max(\tau_i)]$.

$\mathbf{x}_j(t)$ is also written as

(14) $$\mathbf{x}_j(t) = \sum_{l=1}^{L} \mathbf{v}(\theta_{jl}) R_{jl} e^{i\varphi_{jl}(t)} s(t - \tau_{jl})$$

where

(15) $$\varphi_{jl} = 2\pi\pi(f_d Cos\theta_{jl} - f\tau_{jl})$$

Further, all multipaths from the mobile arrive at the base station array uniformly within $\pm\Delta$ around the mean angle of the arrival θ_j.

Further, assuming the signals are narrowband, the complex baseband signal vector can be written as

(16) $$\mathbf{x}_j(t) \approx s_j(t - \tau_0)\left(\sum_{l=1}^{L} \mathbf{v}(\theta_j) R_j e^{i\varphi_l(t)}\right)$$

Further letting

(17) $$\mathbf{a}_j(t) = \sum_{l=1}^{L} \mathbf{v}(\theta_j) R_{jl} e^{i\varphi_{jl}(t)}$$

be an Array Response vector for L number of coherent multipaths corresponding to j'th source.

Use of the conventional algorithms like MUSIC leads to the estimation of the same direction

θ_j, since all the L multipaths corresponding to that source are coherent. Hence θ_{jl} may be replaced by θ_j.

In such a case the above equation can be approximated as

(18) $$\mathbf{a}_j(t) \approx \mathbf{v}(\theta_j)\beta_j, \quad where \quad \beta_j = \sum_{l=1}^{L} R_{jl} e^{i\varphi_{jl}(t)}$$

For J-users the signal vector at the array is given by

(19) $$\mathbf{x}(t) = \mathbf{x}_1(t) + \mathbf{x}_2(t) + \ldots\ldots + \mathbf{x}_J(t) = \sum_{j=1}^{J} s_j(t)\mathbf{a}_j + \mathbf{n}(t)$$

n(t) is the noise vector (column), noise is assumed to be spatially and temporally white.

(20) $$\begin{aligned}\mathbf{R} &= E[\mathbf{y}(t)\mathbf{y}(t)^H] \\ &= \mathbf{A\Phi A}^H + \sigma^2\mathbf{I}\end{aligned}$$

$\mathbf{R}$ is a correlation matrix. It is obtained from the outer-products of the received signal+noise vectors $\mathbf{x}(t)$ and averaged over several snapshots.

Averaging is denoted by E.

H denotes the 'Hermitian(transpose and conjugate).

$\mathbf{A}$ is the matrix of size NxJ (N-antennas, J-sources);

σ^2 is the noise variance,

$\mathbf{I}$ is the NxN identity matrix and

(21) $\Phi = E[\mathbf{s}(t)\mathbf{s}(t)^H]$ is the JxJ source covariance matrix.

s(t) is baseband signal powers lying along the diagonal elements of Φ.

It's size is JxJ, where J is the number of sources. Noise covariance is given by

(22) $$E[\mathbf{n}(t)\mathbf{n}^H(t)] = \sigma_n^2\mathbf{I}$$

So, letting the eigenvalues of $\mathbf{R}$, be arranged in descending order, and denoted by $\lambda_1 \geq \lambda_2 \geq ,... \geq \lambda_N$, as well as letting $\mathbf{u}_1,...,\mathbf{u}_N$ be the corresponding eigenvectors, where the eigenvalues λ_n are given by

(23) $$\lambda_n = \begin{cases} v_n + \sigma^2 \, for\ n = 1,...K \\ \sigma^2 \, for\ n = K+1,...N \end{cases}$$

$\mathbf{U}_s = [\mathbf{u}_1,...,\mathbf{u}_J]$ may be considered as the signal subspace, and

$\mathbf{U}_n = [\mathbf{u}_{J+1},...,\mathbf{u}_N]$ may be considered as the noise subspace.

$\mathbf{U}_n\mathbf{v}(\theta_j) = 0$ for $j = 1,...,J$, and elsewhere $\mathbf{U}_n\mathbf{v}(\theta_j) \neq 0$.

The embodiment assumes that the subspace spanned by the estimated eigenvectors $\mathbf{U}_s = [\mathbf{u}_1,...,\mathbf{u}_J]$ corresponds to the space spanned by the J-true source directions. It should be noted that j-th eigenvector corresponds to L-coherent multipaths due to the j-th source. Estimation of the j-th source direction θ_j can be done by any well known method like MUSIC. The embodiment

searches within the direction range $\theta_j \pm \Delta$, centered on the determined DOA of the j-th source θ_j, to determine the L multipaths for j-th source.

Thus, signal vector $\mathbf{x}_j(t)$ can be considered as

$$\mathbf{x}_j(t) = [\mathbf{v}(\theta_{j1}) \quad \mathbf{v}(\theta_{j2}) \quad \ldots \quad \mathbf{v}(\theta_{jL})]\tilde{\mathbf{s}}_{jl}$$

(24) $\quad$ *where* $\quad \tilde{\mathbf{s}}_{jl} = R_{jl} e^{i2\pi(f_d \cos\theta_{jl} - f\tau_{jl})} s_j(t - \tau_{jl})$

(25) $\quad \mathbf{x}_j(t) \in span\{\mathbf{v}(\theta_{j1}),\ldots\mathbf{v}(\theta_{jL})\}$.

Thus, as illustrated in **Fig. 3**, the process for searching for the set $\Theta_j = (\theta_{j1}, \theta_{j2}, \ldots, \theta_{jL})$ where $\theta_{jl} \in (\theta_j \pm \Delta), l = 1,2,\ldots,L$, starts with the process first selecting a trial direction set $\Theta = (\theta_{jk_l}, \theta_{jk_2}, \ldots, \theta_{jk_L})$ for L multipaths of the jth source, block **302**.

The processing of finding $\Theta = \Theta_j$, that is, $(\theta_{jk_l}, \theta_{jk_2}, \ldots, \theta_{jk_L}) = (\theta_{j1}, \theta_{j2}, \ldots, \theta_{jL})$ is explained as follows.

First, the process selects randomly a set $(\theta_{jk_l}, \theta_{jk_2}, \ldots, \theta_{jk_L})$

Second, the process forms the following matrix

$$\mathbf{D}(\theta) = [\mathbf{v}(\theta_{jk_1}) \quad \mathbf{v}(\theta_{jk_2}) \quad \ldots \quad \mathbf{v}(\theta_{jk_L}) \quad \mathbf{u}_j] \quad where \quad \theta_{jk_l} \in (\theta_j \pm \Delta)$$

Third, the process computes the following relationships

(26) $\quad r_{11} = \| \mathbf{v}(\theta_{jk_1}) \|_2$, and

(27) $\quad \mathbf{q}_1 = \dfrac{\mathbf{v}(\theta_{jk_1})}{r_{11}}$, block **304**.

$\|\cdot\|_2$ is the 2-norm of the vector.

Similarly, for following coefficients and vectors, the process computes

(28) $\quad r_{il} = \mathbf{q}_i^H \mathbf{v}(\theta_{jk_l}); \qquad 1 \le i \le l-1; \qquad l = 2,\ldots,L+1$

(29) $\quad r_{ll} = \| \mathbf{v}(\theta_{jk_l}) - \sum_{i=1}^{l-1} r_{il}\mathbf{q}_i \|_2, \quad l = 2,\ldots,L+1$, and

(30) $\mathbf{q}_l = [\mathbf{v}(\theta_{jk_l}) - \sum_{i=1}^{l-1} r_{il}\mathbf{q}_i] / r_{ll}$, $l = 2,\ldots,L+1$, block **306**. The computation is performed until *l=L+1,* that is obtaining $r_{L+1,L+1}$.

Then, at block **308**, the process computes the function

(31) $B(\Theta_j) = \dfrac{1}{r_{L+1,L+1}}$.

If the result of $B(\Theta_j)$ yields a "new" set of peak values, the trial direction set Θ is set as the directions of arrival of the *L* strong multipaths.

At block **310**, a determination is made whether the process is to be repeated. The number of trials to repeat may be predetermined, for operating efficiency, or based on a predetermined threshold of diminishing marginal improvements.

If another trial set is to be evaluated, the process returns to block **302**.

Eventually, the criteria to terminate estimation process is met, and the process terminates. The then current estimates of the *L* multipath directions $\hat{\Theta}_j = \{\hat{\theta}_{j1},\ldots,\hat{\theta}_{jL}\}$ are used to obtain the combined signal $z(t)$ (See Figure.1) as

(32) $$z(t) = \sum_{l=1}^{L}\sum_{n=1}^{N} x_n w_{nl} \quad \text{where } w_{nl} = e^{-ikd(n-1)\cos\hat{\theta}_{jl}}$$

In other words, after estimation of the *L*-multipath directional set in DOA Estimation Unit 112 for *j*-th source, the received signals at the antenna elements are appropriately weighted as given in the equation (32).

This process is repeated for all *J* signal sources received at the antenna elements with each source signal weighted by coefficients corresponding to *L*-multipaths.

Referring now to **Fig. 4**, a block diagram illustrating an example computer system suitable for hosting a software implementation of the DOA Estimation unit

112 and Beamforming unit **114**, is shown. As illustrated, computing device **400** includes one or more processors **402**, system memory **404**, mass storage devices **406**, other I/O devices **408** and network communication interface **410**, coupled to each other via system bus **412** as shown.

Processor **402** is employed to execute software implementations of DOA Estimation **112** and/or Beamforming **114**. Processor **402** may be any one of a number of processors known in the art or to be designed. Examples of suitable processors include but are not limited microprocessors available from Intel Corp of Santa Clara, CA.

System memory **404** is employed to store working copies of Estimation **112** and/or Beamforming **114** and operating system services. System memory **404** may be Dynamic Random Access Memory (DRAM), Synchronous DRAM (SDRAM) or other memory devices of the like.

Mass storage devices **406** are employed to persistently store data, including e.g. a persistent copy of Estimation **112** and/or Beamforming **114**. Examples of mass storage devices **406** include but are not limited to hard disks, CDROM, DVDROM, and so forth.

Other I/O devices **408** are employed to facilitate other aspects of input/output. Examples of other I/O devices **408** include but are not limited to keypads, cursor control, video display and so forth.

Network communication interface **410** is employed to facilitate network communication with other devices. Network communication interface **410** may be wired based or wireless. In various embodiments, network communication interface **410** may support any one of a wide range of networking protocols.

In alternate embodiments, computing system may be a multi-processor systems, a cluster of networked computers, including an array of massively parallel computing nodes.

Accordingly, various novel methods and apparatus for determining the DOA of *J* signals, and/or determining the DOA of *L* multipaths of *J* signals, the *J* signals being wireless transmitted by their sources.

While the present invention has been described in terms of the foregoing embodiments, those skilled in the art will recognize that the invention is not limited to the embodiments described. Other embodiments may be practiced with modification and alteration within the spirit and scope of the appended claims. Accordingly, the description is to be regarded as illustrative instead of restrictive.